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Two experiments with rotating magnetic field

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Abstract

Two student experiments involving a rotating magnetic field are described. The first experiment consists of measurements of the rotational speed of an induction motor versus its load. The second is a determination of the torque on a conductor as a function of the frequency of rotation of the magnetic field. The experiments may become a useful addition to those published earlier.

1. Introduction

The interaction of a conductor with a rotating magnetic field is an important topic of electromagnetic theory. It has many applications; first, in induction motors. A rotating magnetic field is used for contactless measurements of electrical conductivity of metals and semiconductors. For very low resistivities, for example for pure metals at liquid helium temperatures, this technique has important advantages over other contactless methods. From an educational point of view, the basic question of this topic is the frequency dependence of the torque on a conductor by a rotating magnetic field.

Several papers have been devoted to the interaction of a rotating conductor with a dc magnetic field. Doyle and Gibson (1979) described a classroom demonstration of retarding and repulsive forces acting on a permanent magnet put near a rotating conductor. In this demonstration, a lightweight cylindrical bar magnet is hung on a quadrifilar suspension with one of the magnet poles near the face of an aluminium disc driven by a variable-speed motor. Wiederick *et al* (1987) have proposed an experiment involving a magnetic brake of a thin aluminium disc freely rotating between the pole pieces of an electromagnet. The braking force is proportional to the magnetic field squared, in accordance with theory. Saslow (1987) considered a number of examples illustrating the electromechanical implications of Faraday's law, including eddy current brakes and induction motors. Marcuso *et al* (1991a) computed the torque on a moving conductor under the influence of a localized magnetic field. The authors confirmed the calculations by measurements of the deceleration of aluminium and copper discs freely rotating in a localized, non-uniform magnetic field (Marcuso *et al* 1991b).

The first experiment described below includes measurements of the rotational speed of an induction motor versus its load. Any increase of the load causes a decrease of the speed of rotation and hence increases the eddy currents in the rotor and the driving torque. At any load, the speed of rotation matches it to supply the necessary torque. The induction motor is an excellent example for the demonstration of eddy currents in a conductor and Lenz's law, as well as the energy conservation in electromagnetic phenomena.

The second experiment consists in measuring the torque applied to a cylindrical conductor as a function of the frequency of rotation of the magnetic field. In contrast to all the experiments mentioned above, the results are obtained also in a frequency range beyond the quasi-static limit. In addition, this experiment illustrates one of the contactless methods of measuring the electrical conductivity of metals and semiconductors.

2. Induction motor

In an induction motor, a two- or three-phase ac current supplied to the stator windings produces a rotating magnetic field, inducing eddy currents in the rotor (e.g., Smith 1984). The stator field rotates at a frequency f_0 determined by the frequency of the current and the number of poles. According to Lenz's law, the interaction between the currents induced in the rotor and the rotating magnetic field produces a torque. The rotor starts to rotate at a frequency f somewhat lower than the frequency of the rotating magnetic field. The lag of the rotor relative to the rotating magnetic field necessary for the motor action is called the slip $s = (f_0 - f)/f_0$.

The experiment consists of determinations of the slip of an induction motor versus its load. A dc generator with independent excitation serves as the load (figure 1). Changes of the output current of the generator cause changes of the torque necessary to operate it. This allows one to vary the load of the induction motor. The changes of the load equal the changes of the dc electric power produced by the generator.

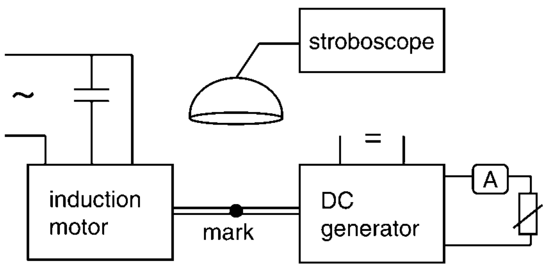

Figure 1. Set-up to measure the rotational frequency of the induction motor versus its load.

The rotating magnetic field is created by ac currents in the stator coils of the motor, shifted by 90° by means of a suitable capacitor. In our case, the rotational frequency of the magnetic field is $f_0 = 25$ revolutions s^{-1}. A flexible plastic tube fits the shafts of the motor and of the generator. A mark on the tube serves to determine the rotational frequency of the rotor by means of a stroboscope. The resolution of the stroboscope was 0.1 Hz. To reduce the error caused by the insufficient resolution, the stroboscope's frequency is set to be five times higher than the frequency of the rotation, f. The resolution thus becomes 0.02 Hz. The frequency of the rotation may be also determined using a small permanent magnet pasted to the shaft of the motor, a coil sensing fast changes of the magnetic flux, and a frequency meter.

The calculations of the electric power produced by the generator should take into account the power dissipated in its rotor winding. The total electric power is $P = IE$, where I is the current and E is the EMF of the winding. A variable load resistor regulates the current, and a meter measures it. The EMF generated by the rotor should be measured when $I = 0$; in addition, it is proportional to the frequency of rotation. Thus the power is $P = IE_1 f/f_1$, where E_1 is the EMF when the frequency of rotation is f_1.

The results obtained (figure 2) show a linear dependence of the slip versus the load, in agreement with theory. Here the slip at the zero output power of the generator is too large. This is due to poor matching of the motor and the generator.

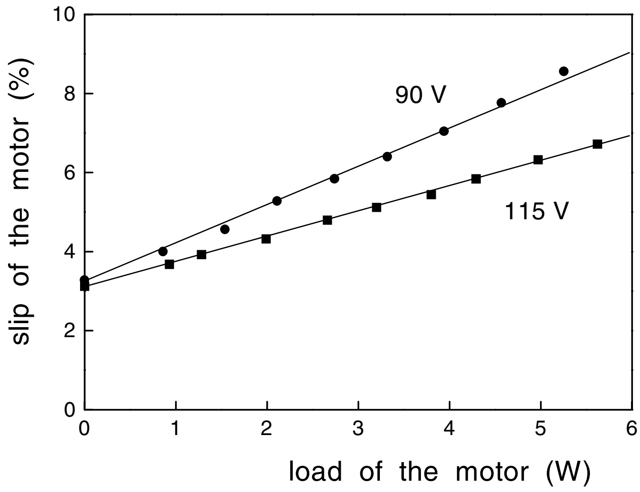

Figure 2. The rotational frequency of the induction motor versus load for two feeding voltages.

3. Torque versus frequency and the electrical resistivity of cylindrical samples

The torque on a conductor caused by a rotating magnetic field depends on the magnitude and the frequency of the rotation and on the shape and electrical resistivity of the sample. At low frequencies, where the size of the conductor remains smaller than the skin depth, the torque is proportional to the frequency of the rotation of the magnetic field. At higher frequencies the torque reaches a maximum and then decreases because of the skin effect. Due to the skin effect, the eddy currents in the conductor become smaller, and a phase shift arises between the EMF induced in the conductor and the current in it.

The torque on a conductor caused by a rotating magnetic field was calculated theoretically. For a spherical sample, the solution may be found, for example, in Landau and Lifshitz (1984). For a cylindrical sample, Batygin and Toptygin (1970) considered the problem. The torque for a unit length of the cylinder is

$$M = -aH^2 \mathrm{Re}\,[kJ_1(ka)/J_0(ka)]/|k|^2$$

where H is the magnetic field, $k^2 = \mathrm{i}\omega\mu_0/\rho$, $\omega = 2\pi f$ is the angular frequency of the rotation, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1} is the permeability of free space, a and ρ are the radius of the cylinder and its electrical resistivity, and J_1 and J_0 are Bessel functions of the first kind.

The argument of the Bessel functions is a complex quantity because $\sqrt{\mathrm{i}} = (1 + \mathrm{i})/\sqrt{2}$. These functions were tabulated (Jahnke and Emde 1945), and one can evaluate the theoretical dependence of the torque versus the speed of rotation of the magnetic field. A more convenient argument for this dependence is $X = |k^2|a^2/2 = \omega\mu_0 a^2/2\rho = (a/\delta)^2$, where δ is the skin depth.

It is easy to see that the maximum torque M_0 does not depend on the resistivity of the sample, while the relative torque $Y = M/M_0$ is a universal function of X. Therefore, data for cylindrical samples of various thicknesses and resistivities can be displayed together and compared with the theoretical curve $Y(X)$. By the use of this dependence one avoids determinations of the properties of the suspension.

In our set-up, two pairs of coils create magnetic fields in two perpendicular directions (figure 3). The ac currents through the coils are shifted by 90°, which leads to the rotation of the magnetic field. A multifrequency synthesizer, HP 8904A, supplies the currents. The device has two channels, the frequencies and phases set independently. In our case the phases are set to be 0 and 90°. The frequency of the rotation equals the frequency of the currents.

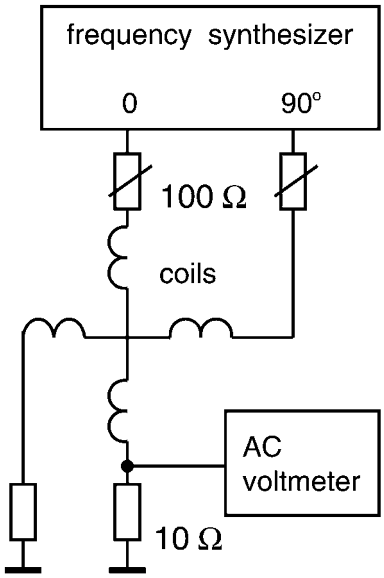

Figure 3. Electric circuit to create a rotating magnetic field of various frequencies.

A switch (not shown in the figure) changes the direction of the current in one pair of the coils and thus reverses the direction of the torque.

The sample is suspended on a copper wire, 0.1 mm in diameter and 40 cm long (figure 4). A small flat mirror is pasted to the lower end of the wire. A laser beam reflected from the mirror falls onto a scale arranged at 1.5 m from the mirror. The readout is taken as the difference between the deflections corresponding to the two directions of the torque. The samples are cylinders, 1–1.5 cm in diameter and 10 cm long. The lower end of the sample is located in a region of a strong magnetic field of a permanent magnet. We use a variable gap magnet from PASCO, catalogue number EM-8641. Two resistors are connected in series with each pair of coils, variable resistors of 100 Ω and constant resistors of 10 Ω (1%). The variable resistors allow one to maintain the same feeding currents for various frequencies. An electronic ac voltmeter measures the voltage drop across the constant resistors, which are proportional to the feeding currents.

The measurements are carried out on two samples, copper and an aluminium alloy. To calculate the argument X, one has to know the electrical resistivity of the samples. For the samples employed, the resistivity was previously measured by a contactless method based on determinations of the phase angle of the effective magnetic susceptibility of cylindrical samples in an ac magnetic field (see, e.g., Chambers and Park 1961, Kraftmakher 1991). The resistivity was found to be 1.73×10^{-8} Ω m for copper and 4.04×10^{-8} Ω m for the aluminium alloy (Kraftmakher 2000). Now the experimental data for both samples can be displayed together and superimposed on the theoretical curve. The results are in reasonable agreement with theory (figure 5).

4. Contactless measurements of electrical resistivity

Three main techniques are known for contactless measurements of electrical resistivity (see a review by Delaney and Pippard 1972), namely: (i) determination of the effective magnetic susceptibility of cylindrical samples in an axial ac magnetic field, (ii) observation of the decay of the eddy current in a sample after terminating an external dc magnetic field, and (iii) measurement of a torque on a sample caused by a rotating magnetic field.

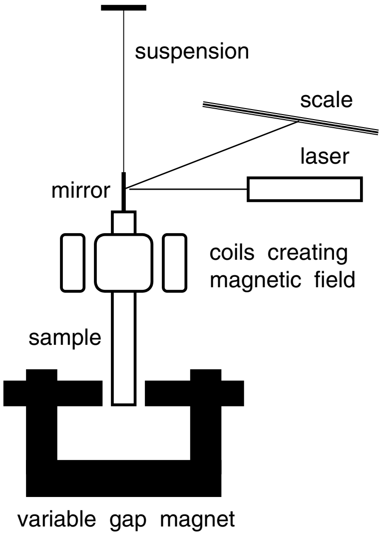

Figure 4. Set-up to measure the torque on a cylindrical conductor caused by a rotating magnetic field.

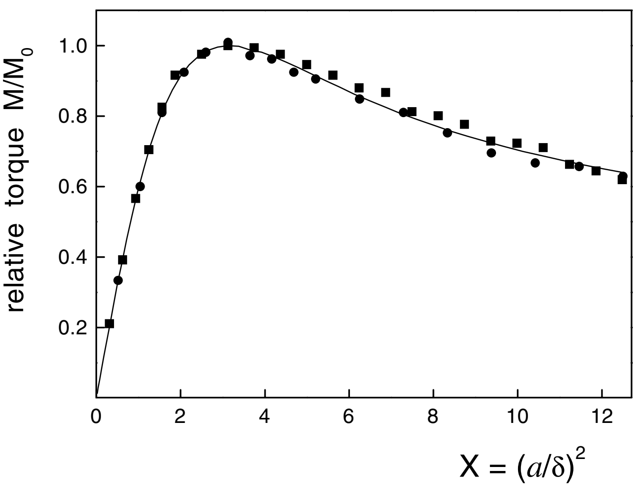

Figure 5. Universal dependence of the relative torque M/M_0 versus $(a/\delta)^2 = \omega\mu_0 a^2/2\rho$: ——denotes the theoretical curve, ● denotes experimental data for copper samples, and ■ denotes experimental data for aluminium alloy samples.

The last method was employed, for example, by Zernov and Sharvin (1959) to measure the resistivity of high-purity tin at liquid helium temperatures. To perform the measurements in the quasi-static limit, the authors used a very low frequency for the rotating magnetic field, 2×10^{-3} Hz. Helmholtz coils fed by a dc current and rotating about a vertical axis created the field. The residual resistivity of the sample was found to be 3.7×10^{-13} Ω m.

The electrical resistivity of a sample can be determined from the frequency of the rotation of the magnetic field that provides the maximum torque (Delaney and Pippard 1972). A more accurate method is a determination of the quantity $a^2 f/\rho$ for which a given value of the relative torque $Y = M/M_0$ is achieved (Zakharov and Kraftmakher 1989).

When measuring very low electrical resistivities, the employment of a rotating magnetic field has definite advantages over other contactless techniques. The maximum torque acting on a sample does not depend on its resistivity. This means that the sensitivity of the method, in contrast to other contactless methods, can be kept constant regardless of the resistivity to be measured; for this purpose it is enough to adjust the rotational frequency of the magnetic field. When measuring very low resistivities with other contactless techniques one has to employ very low frequencies of the magnetic field or observe a very long decay of the eddy currents. In both cases, the voltages to be measured become too small.

Acknowledgment

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